Int. J. Heat Mass Transfer. Vol. 13, pp. 275–286. Pergamon Press 1970. Printed in Great Britain

SIMULTANEOUS MEASUREMENT OF SIX THERMAL PROPERTIES OF A CHARRING PLASTIC

ROBERT C. PFAHL, JR,* and BARRY J. MITCHEL
Avco Corp., Applied Technology Division, Wilmington, Mass., U.S.A.

(*Received* 9 *May* 1969 *and in revised form* 9 *July* 1969)

Abstract—Six thermal properties of a charring carbon–phenolic material are measured in a single experiment. The experiment, performed in two hyperthermal plasma-arc facilities, is described and a heat-transfer model is presented, an extension of the classical fusion model of Stefan. The model is solved using a method of finite differences which has several unique features. The properties are calculated from the experimental measurements using the model and nonlinear regression analysis. The calculated property values are shown to be in good agreement with values from conventional tests. It is demonstrated by comparative calculations that the modified Stefan model with nonlinear-regression properties predicts the transient thermal behavior equally as well as a more sophisticated ablation model with conventionally measured properties.

NOMENCLATURE

C, defined by equation (14) [J/m^3 deg];
c, specific heat [J/kg deg];
ΔH, enthalpy increase during reaction [J/kg];
K, defined by equation (13) [W/m deg];
k, thermal conductivity [W/m deg];
L, initial specimen length [M];
p, property;
R, stretching ratio defined by equation (28);
S, heat sink capacity at cooled surface [J/m^2 deg];
T, temperature [°C];
TC, thermocouple;
t, time [s];
x, spatial ordinate [m];
v, rate of motion of char–virgin interface [m/s];
α, thermal diffusivity [m^2/s];
ν, number of dimensions less one;
ξ, transformed spatial variable;
ρ, material density [kg/m^3];
τ, transformed time variable [s];
ϕ, stretched spatial variable.

Subscripts
c, charred material;
d, decomposition (charring) process;
g, gases formed by pyrolysis of virgin material;
i, $i = c$ or v;
I, interface of decomposition (char front);
B, back surface;
v, virgin material;
F, front surface;
o, initial conditions.

INTRODUCTION

THE AUTHORS have studied the internal behavior of charring ablators with the goal of developing economical, mathematical models which accurately describe thermal behavior. Internal behavior can be considered separately from surface phenomena by using an internal temperature near the heated, ablating surface as one of the boundary conditions in the model. The resulting problem is still a complex heat and mass transfer problem involving the charring or degradation of a polymeric resin and subsequent high-temperature reactions with the reinforcing agent. This problem still has not been completely modeled. Some question

* Now at Western Electric Engineering Research Center, Princeton, N.J., U.S.A.

exists concerning the degree to which a model can be simplified without losing the ability to predict thermal behavior with acceptable accuracy. This question can be answered for each particular material only by comparing predictions with experiment for several models. The simplest acceptable model should be used for two reasons: (1) to simplify the solution of the mathematical model and (2) to minimize the number of thermal properties that must be measured.

Conventionally one measures each required property in a separate "property measurement" test. When using the conventional approach to measure properties for a charring ablator model, (1) it is not always possible to design tests to measure properties under the desired thermal conditions, and (2) there are some properties which are difficult to measure. Consider an example of each situation.

(1) The chemical structure of certain charring ablators depends both on temperature and on the rate of heating; thus, the structure of a material specimen charred slowly in an oven may be different from another specimen charred rapidly during reentry. It is therefore reasonable to expect the thermal conductivities at a given temperature to be different for the two specimens since their chemical compositions differ; yet it is customary practice to determine high-temperature thermal conductivities by performing measurements on specimens which have been oven-charred, and to use these values to describe the behavior of the material when it is rapidly charred. (2) Properties such as the specific heat of the gaseous products of decomposition and reaction-rate constants are difficult to measure, and the current practice is to estimate them by reasonable numbers which give good agreement between theory and experiment.

These problems can be avoided by simultaneously measuring all desired properties using nonlinear regression* to calculate the properties. Beck was the first author to apply the procedure to thermal problems in a series of papers [1–4]. The mathematical details of the technique and several variants have been discussed elsewhere by the senior author [5]; only an outline of nonlinear regression will be presented here.

Consider a thermal system described by a mathematical model expressing the relationship between a theoretical dependent variable, T, several independent variables, x and t, and several properties, $p_1, p_2, \ldots p_n$. In particular, let T be temperature, x be distance and t be time. Symbolically the model† is represented as

$$T = T(x, t; p_1, p_2, \ldots, p_n). \tag{1}$$

Consider an experiment in which the experimental dependent variable, E, is measured at r discrete values of the independent variables, x and t. These r measurements of the experimental temperature are represented as

$$E_i = E(x_i, t_i) \quad \text{for} \quad i = 1, 2, \ldots, r. \tag{2}$$

Nonlinear regression is the process of determining the value of the n properties which will minimize the sum-of-squares function

$$F = \sum_{i=1}^{r} [E(x_i, t_i) - T(x_i, t_i; p_1, p_2, \ldots p_n)]^2. \tag{3}$$

$$\text{where } r \geqslant n$$

Thus, the procedure determines the property values which will minimize the difference in a least-squares sense between theory and experiment for all available data. Equation (3) is normally minimized by an iterative procedure, which requires the solution of equation (1) $n + 1$ times for each iteration [5]. The accuracy of the method is dependent on the design of the experiment. The experiment should be designed by analytical, optimizing techniques [1–4].

* Nonlinear regression is also called nonlinear least-squares or nonlinear estimation.

† No restrictions are placed on the form of the model or the techniques used to solve the model. "Nonlinear" regression implies that the dependent variable is a nonlinear function of the properties. In this article we treat a model consisting of two coupled nonlinear partial differential equations.

In the following sections the authors describe the tests conducted to evaluate the behavior of a carbon–phenolic ablator, introduce a particular model selected to describe the thermal behavior of the ablator, present the properties for the model as determined by nonlinear regression, and compare these properties with conventionally measured properties, demonstrating successful application of the nonlinear regression techniques to a complex heat-transfer problem.

EXPERIMENT

Tests were conducted in two hyperthermal testing facilities to observe the thermal behavior of a carbon–phenolic ablator. These tests were part of other material evaluation efforts by colleagues of the authors. The authors discovered, in spite of the fact these tests had not been designed to measure properties, the experimental data could be analyzed successfully by nonlinear regression. However, the tests were not of optimum design.

The experiment consisted of four tests; three were conducted in the Avco Space System Division's Orbital Vehicle Reentry Simulator (OVERS) Facility [6]. This facility consists of an electric arc heater with a supersonic nozzle, an evacuated 0·6 m dia test chamber, a vacuum pumping system, a 500 kW rectified power supply, and associated instrumentation and data reduction facilities. The OVERS arc is a steady-state device capable of operating within an enthalpy range of 5·8–46·5 MJ/kg for simulated air. The heated gas impinges on the plane surface of a cylindrical specimen. The remaining test was conducted in the Avco Space Systems Division's Ten Megawatt Arc (10 MW) Facility [6]. This arc facility consists of a 0·1 m dia spherical plenum chamber into which four individual arc heads exhaust. The four arcs are mounted in a common plane and are spaced equally around the periphery of the plenum chamber. The heated air mixes in the plenum chamber and exhausts through a supersonic nozzle in a direction perpendicular to the plane of the four arc heads. The plasma then flows through the center of a cylindrical pipe specimen constructed from the material under evaluation.

Figure 1 illustrates the cylindrical specimen design of the three samples tested in the OVERS Facility; Fig. 2 illustrates the specimen design of the one sample tested in the 10 MW Facility. Table 1 tabulates the dimensions and environmental conditions for each sample. The heat fluxes ranged from 0·45 to 15·8 MW/m^2. The carbon–phenolic material consisted of layers of carbon cloth impregnated with phenolic resin; therefore, the material was anisotropic. As noted in Table 1, the angle between the direction of heat flow and a normal to the plane of the cloth layer differed in the two specimen designs; thus, the thermal conductivity can be expected to be somewhat different for the two experimental configurations. Because of the different experimental configurations, the heat transfer in the OVERS tests was one-dimensional in a Cartesian coordinate system whereas in the 10 MW the heat transfer was one-dimensional in a cylindrical system. All temperature measurements were made using chromel–alumel thermocouples installed parallel to the heated surfaces to minimize experimental error [7] and were recorded on magnetic tape. Values of the temperature at desired times were obtained on 80 column punch cards by using a computerized data reduction facility.

MODEL

The model adopted to describe the thermal behavior of the carbon–phenolic material is the classical fusion model of Stefan [8], extended by Grosh [9] and Barriault and Yos [10], in which a virgin material decomposes endothermically at a specified temperature to form a char matrix and a gas. The evolved gas is assumed to remain in local thermal equilibrium as it transpires through the matrix toward the heated surface. Figure 3 illustrates the relevant geometry. The region of interest extends from the forward thermocouple, which supplies a

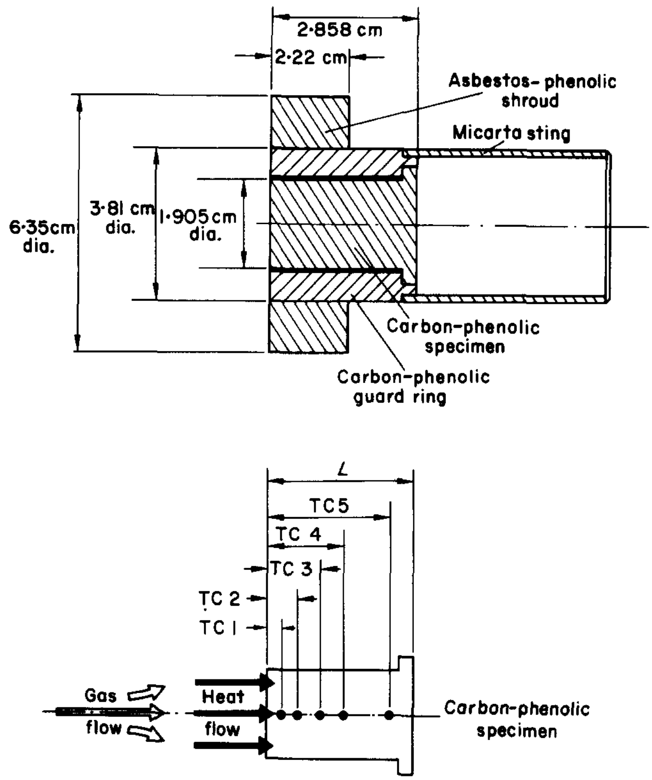

FIG. 1. OVERS specimen design.

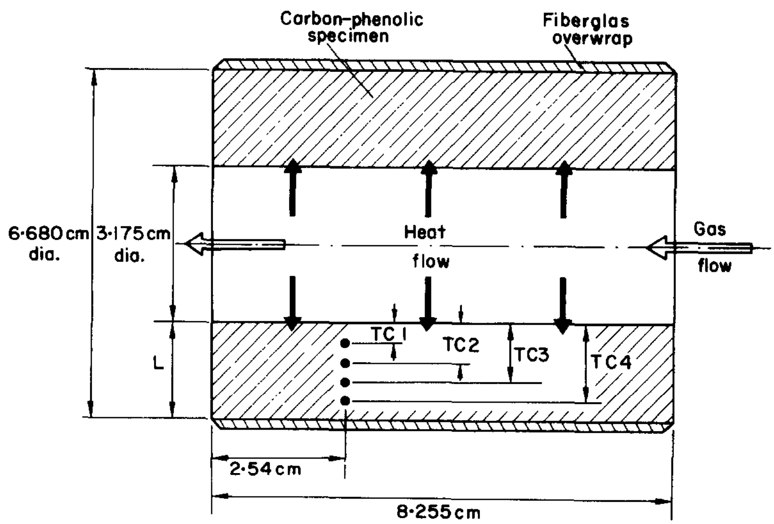

FIG. 2. 10 MW specimen design.

Table 1. Test specimen dimensions and environmental conditions

Specimen	Facility	Cold wall heat flux [MW/m²]	Exposure time [s]	Fabric layup angle	Inner radius [cm]	L [cm]	TC #1 [cm]	TC #2 [cm]	TC #3 [cm]	TC #4 [cm]	TC #5 [cm]
1	OVERS	0·45	240·0	0°	—	2·858	0·254	0·502	1·007	1·53	2·54
2	OVERS	1·03	180·0	0°	—	2·858	—	0·513	1·020	1·53	2·53
3	OVERS	4·11	120·0	0°	—	2·858	—	—	1·020	1·53	2·53
4	10 MW	15·8	4·12	20°	1·59	1·75	0·254	0·511	0·760	1·02	—

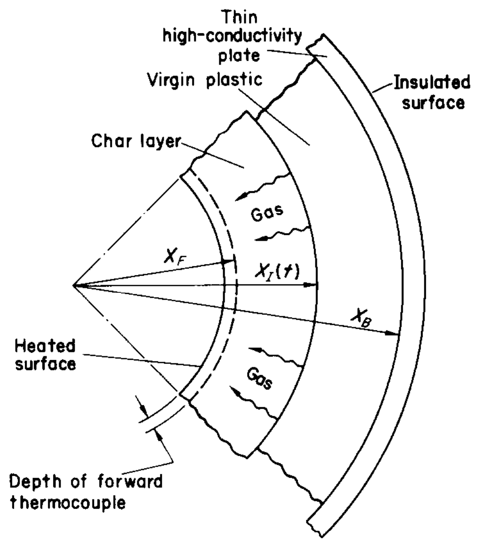

FIG. 3. Geometry for the model.

variable-temperature boundary condition, to the rear of the virgin material, where a thin, high-conductivity plate provides a calorimeter-type boundary condition. The following equations de cribe the flow of heat:

$$T(x, 0) = T_0(x), \quad t = 0 \tag{4}$$

$$T(x_F, t) = T_F(t), \quad x = x_F \tag{5}$$

$$\frac{\partial}{\partial x}\left(Kc\frac{\partial Tc}{\partial x}\right) + C_g v_I \frac{\partial Tc}{\partial x} = C_c \frac{\partial Tc}{\partial t}, \quad x_F < x < x_I(t) \tag{6}$$

$$\rho_v \Delta H_d v_I = k_v \frac{\partial Tv}{\partial x} - k_c \frac{\partial Tc}{\partial x}, \quad x = x_I(t) \tag{7}$$

$$x_I(t) = x_I(0) + \int_0^t v_I(\tau)\, d\tau \tag{8}$$

$$v_I \geqslant 0 \tag{9}$$

$$T(x_I, t) \leqslant T_d \tag{10}$$

$$\frac{\partial}{\partial x}\left(K_v \frac{\partial Tv}{\partial x}\right) = C_v \frac{\partial Tv}{\partial t}, \quad x_I(t) < x < x_B \tag{11}$$

$$-k_v \frac{\partial Tv}{\partial x} = S\frac{\partial Tv}{\partial t}, \quad x = x_B. \tag{12}$$

The equations are generalized to one-dimensional rotationally symmetrical flow in $(\nu + 1)$ space by defining K and C as follows for each region:

$$K_i = k_i(x/x_F)^\nu \tag{13}$$

$$C_i = \rho_i c_i (x/x_F)^\nu \quad \text{where} \quad i = c \text{ or } v \tag{14}$$

and for the gas,

$$C_g = (\rho_v - \rho_c)\, c_g [x_I(t)/x_F]^\nu. \tag{15}$$

In the four experiments reported in this article, the entire material was initially in the virgin state, although this is not a restriction of the model. The initial temperature distribution, equation (4), is obtained by linear interpolation between the initial experimental thermocouple readings. Initially only equations (4), (5), (11) and (12) apply, with $x_I \leqslant x_F$. From the time when the front-boundary temperature first exceeds the decomposition temperature until the entire material has charred, equations (5)–(12) are solved simultaneously. When the material is completely charred, only equations (5), (6) and (12) apply with $x_I = x_B$. Since, unlike fusion, the decomposition process is irreversible, the model has a provision for temporary halting of the char–virgin interface anywhere between the boundaries. For such time intervals, the interface temperature varies in such a way that equation (7) is satisfied for $v_I = 0$. For the two experimental configurations presented in the previous section, the rear surface may be assumed to be adiabatic and therefore $S = 0$ in equation (12).

The system of equations (4)–(12) is transformed in two steps to facilitate its solution by finite-difference methods. The first step is to apply the Murray–Landis [11] transformation:

$$\xi_c = (x - x_F)/(x_I - x_F), \quad x_F \leqslant x \leqslant x_I \tag{16}$$

$$\xi_v = (x - x_I)/(x_B - x_I), \quad x_I \leqslant x \leqslant x_B \tag{17}$$

$$\tau = t. \tag{18}$$

The second step in the transformation is a nonlinear stretching of the variable ξ so that more grid points can be put into regions of higher temperature gradients without the need for unsymmetrical difference approximations for the gradients. The general form of the stretched ordinate is $\phi = \phi(\xi)$, with $\phi(0) = 0$ and $\phi(1) = 1$ for convenience. Application of the two-step transformation to equations (4)–(12) yields:

$$T(\phi, 0) = T_0(\phi), \quad \tau = 0 \tag{19}$$

$$T(0, \tau) = T_F(t), \quad \phi_c \text{ or } \phi_v = 0 \tag{20}$$

$$\frac{\phi_c'}{(x_I - x_F)^2}\frac{\partial}{\partial \phi_c}\left(K_c \phi_c' \frac{\partial T_c}{\partial \phi_c}\right) + v_I \frac{(C_g + \xi C_c)}{x_I - x_F}\phi_c' \times \frac{\partial T_c}{\partial \phi_c} = C_c \frac{\partial T_c}{\partial \tau}, \quad 0 < \phi_c < 1 \tag{21}$$

$$\rho_v \Delta H_d v_I = \frac{k_v \phi_v'}{x_B - x_I}\frac{\partial T_c}{\partial \phi_v}\bigg|_{\phi_v = 0} - \frac{k_c \phi_c'}{x_I - x_F}\frac{\partial T_c}{\partial \phi_c}\bigg|_{\phi_c = 1} \tag{22}$$

$$x_I(\tau) = x_I(0) + \int_0^\tau v_I(\tau)\,\mathrm{d}\tau \tag{23}$$

$$v_I \geqslant 0 \tag{24}$$

$$T_c(1, \tau) = T_v(0, \tau) \leqslant T_d \tag{25}$$

$$\frac{\phi_v'}{(x_B - x_I)^2}\frac{\partial}{\partial \phi_v}\left(K_v \phi_v' \frac{\partial T_v}{\partial \phi_v}\right) + \frac{(1 - \xi_v)v_I}{x_B - x_I} C_v \phi_v' \frac{\partial T_v}{\partial \tau}, \quad 0 < \phi_v < 1 \tag{26}$$

$$\frac{-k_v \phi_v'}{x_B - x_I}\frac{\partial T_v}{\partial \phi_v} = S\frac{\partial T_v}{\partial \tau}, \quad \phi_v = 1 \tag{27}$$

in which $\phi' = \mathrm{d}\phi/\mathrm{d}\xi$ is a function of ξ. The particular form of ϕ used in the model is given implicitly by the quadratic:

$$\xi_i = [2\phi_i + (R_i - 1)\phi_i^2]/[R_i + 1], \quad \text{where} \quad i = c \text{ or } v \tag{28}$$

R equals the ratio of the extremal stretching rates, $\phi'(0)/\phi'(1)$. Therefore, when $R > 1$, grid points are more dense in the original space near $\xi = 0$ than near $\xi = 1$.

The method of numerical solution is similar to that of Swan and Pittman [12], with the addition of the nonlinear grid system described above and the following refinements:

1. An implicit rather than explicit differencing scheme is used to improve stability and to allow greater freedom in the choice of grid spacing and time step. The degree of implicity may be varied in the digital computer program. Crank–Nicholson [13] differences were used for the present study.
2. The numbers of grid points in the char and virgin regions are increased and decreased, respectively, as the char front moves through the material, in such a way that the grid Fourier numbers, $\alpha_i\,\Delta t/(\Delta x)^2$, in these two regions never differ by more than a factor of 2. Cubic interpolation from four neighboring grid points is used to compute temperatures at newly created grid points in the char.
3. As a result of the variable number of grid points, the initial char depth can be zero, unlike references [11] and [12]. When $T_F(t)$ first exceeds T_d, the char–virgin interface speed

v_I and position x_I are solved from difference equations representing (22) and (23). At this time, the growing char layer is bounded by grid points at known temperatures $T_F(t)$ and $T_I = T_d$, but has no internal grid points. This procedure is continued until the char grid spacing $\Delta x_c = x_I(t) - x_F$ violates the grid Fourier-number balance, and an internal grid point is created in the char as explained above. A similar procedure has been developed for a heat-flux boundary condition at x_F.

The transformed system of equations (19)–(27) has been programmed in the Fortran II language for the IBM 7094 digital computer. The program selects the time steps and the times at which the numbers of grid points should be changed to satisfy the grid Fourier-number balance mentioned above. The procedure for selection of time steps is to begin with a maximum grid Fourier number, $\alpha_i \Delta t/(\Delta x)^2$ equal to 0·5, and to increase the grid Fourier number by 10 per cent after each time step.* During time intervals when special accuracy is essential, such as near the starting or stopping of charring, the time step is reduced temporarily using other criteria.

The programmed heat-transfer solution, while complete in itself, is incorporated as a subroutine of the programmed nonlinear regression calculations. This complex subroutine is represented by equation (1) in the nonlinear regression analysis. Equation (1) is solved $n + 1$ times at each iteration of the regression calculations. The automatic process for choosing grid spacings and time steps is necessary since the range of properties selected during the iterations cannot be known in advance. One feature of the combined nonlinear regression–heat transfer program is directed acceptance of the 80 column punch cards provided by the computerized data reduction facility. Since the accuracy of the nonlinear regression calculations improves by using all of the large volume of available data, compatibility of these systems is essential.

* In a recent note, Weber [19] has suggested a similar procedure for improving the accuracy of Crank–Nicolson solutions. His note provides analytical justification for the procedure.

PROPERTY DETERMINATION BY NONLINEAR REGRESSION

If the thermal properties are assumed to be independent of temperature, the adopted charring ablator model described by equations (4)–(15) involves nine properties; k_v, c_v, ρ_v, k_c, c_c, ρ_c, c_g, ΔH_d. Careful examination of the set of equations forming the model reveals only seven independent parameters. Since for the experiments performed, $S = 0$ in equation (12), this number was reduced further to six. In other words, if the model describes the thermal behavior of a certain material, only six quantities are necessary to characterize the behavior of that material. These six quantities or thermal parameters are functions only of the nine properties. The selection of the six parameters is not unique, just as the selection of dimensionless groups is not unique in dimensional analysis. If the experiments had been designed specifically for nonlinear regression analysis, a calorimeter plate would have been provided at the rear so that $S > 0$; the plate would have made possible the determination of one additional parameter [14]. The authors chose to use the values of ρ_v, ρ_c and c_v (which were measured by conventional tests) as known constants. This choice allowed the remaining six properties (k_v, k_c, c_c, c_g, T_d, ΔH_d) to be the six parameters determined by nonlinear regression. They were determined from the data of the three tests conducted in the OVERS facility by using the combined nonlinear regression–heat transfer computer program. Property values were determined for each specimen; these three sets of values were then linearly averaged to determine the set of values presented in the following section.

COMPARISON WITH CONVENTIONAL MEASUREMENTS

To demonstrate the utility and accuracy of simultaneous property measurements, nonlinear

Table 2. Thermal properties of the carbon–phenolic ablator

Property	Units	Values determined by nonlinear regression	Conventional measurements
k_v	W/m deg	0·850	0·57–1·04*
k_c	W/m deg	0·751	0·38–1·09†
c_c	kJ/kg deg	1·13	0·887
ΔH_d	MJ/kg virgin material	0·711	0·775
T_d	°C	346·0	—
c_g	kJ/kg deg	−3·29	+1·67
		Values arbitrarily specified based on conventional measurements	
c_v	kJ/kg deg	1·045	1·045
ρ_v	kg/m^3	1440·0	1440·0
ρ_c	kg/m^3	1120·0	960·0–1440·0‖

* Function of temperature-continuous variation with temperature.
† Continuous variation with temperature and charring temperature.
‖ Continuous variation during degradation.

regression results are compared with conventionally measured values. Since the ultimate application of the properties is prediction of transient thermal behavior, the modified Stefan model with nonlinear regression properties must be able to predict thermal behavior as well as accepted ablation models with conventionally measured properties. The authors were fortunate in having available for comparison an earlier study [15] of a colleague, R. S. Bartle. He studied the three tests conducted in the OVERS Facility using the Munson–Spindler [16] charring ablator model. This model differs from the authors' modified Stefan model primarily by using an "nth" order reaction rate equation with continuously variable density as opposed to defining a decomposition temperature, T_d, and two discrete densities. Thus, the property T_d, is not required in the Munson–Spindler model, but two other properties, degradation rate constants, are required. These rate constants are difficult to measure directly; one method is to deduce them simultaneously by thermogravimetric analysis (TGA) of a small specimen.‡

Bartle used conventional thermal conductivity measurements made with a steady-state guard-ring configuration apparatus on oven-charred specimens. The conductivity was measured as a function of both density and temperature. The density and specific heat were also measured by conventional methods on oven-charred specimens. Since ΔH_d and c_g cannot be measured readily by conventional tests, Bartle chose reasonable values based on previous experience to obtain agreement between the theory and experiment.

The property values determined by the nonlinear regression calculations and the conventionally measured values used by Bartle are tabulated in Table 2. Good agreement is noted. The only property for which there is lack of agreement is c_g, the specific heat of the transpiring gas. For all of the tests the calculated transient temperature response of the material is quite insensitive to the value of c_g. This insensitivity indicates that the experimental design is inadequate for measuring c_g. Therefore, little significance can be attached to the value of c_g determined by nonlinear regression. However, the negative value obtained in this case is a physical possibility; it may indicate that an exothermic process is occurring either in the gas phase or between the gas phase and surrounding char matrix. One possible exothermic process would be the deposition of pyrolytic graphite on the char matrix.

‡ Linear regression analysis has been introduced recently into TGA [17, 18].

The nonlinear regression properties were used in the author's model and the conventional properties were used by Bartle in the Munson–Spindler model to predict the thermal response of all four specimens. The experimentally measured temperature at the front thermocouple was used as the forward boundary condition. The environmental conditions and geometry for each specimen are presented in Table 1. Figures 4–7 present the thermal response predicted by the two sets of properties as well as the experimental measurements.*

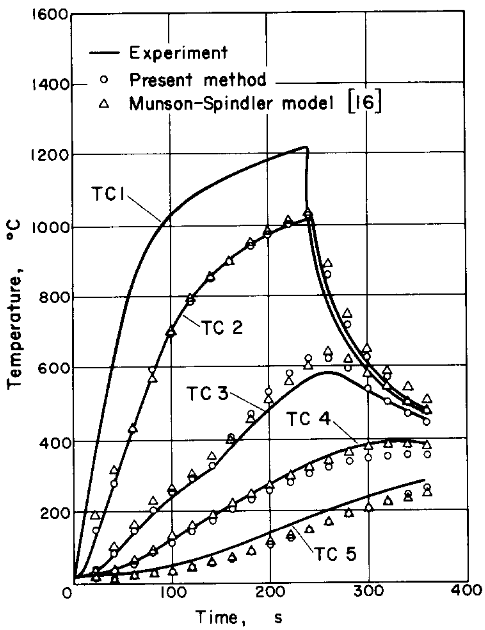

FIG. 4. Temperature histories for specimen 1.

There is good agreement except for thermocouple four of specimen three (Fig. 6). Bartle and the authors believe that the lack of agreement between theory and experiment for this sensor may be due to a thermocouple error. The fact that the two theoretical predictions agree closely supports this hypothesis. Figure 7

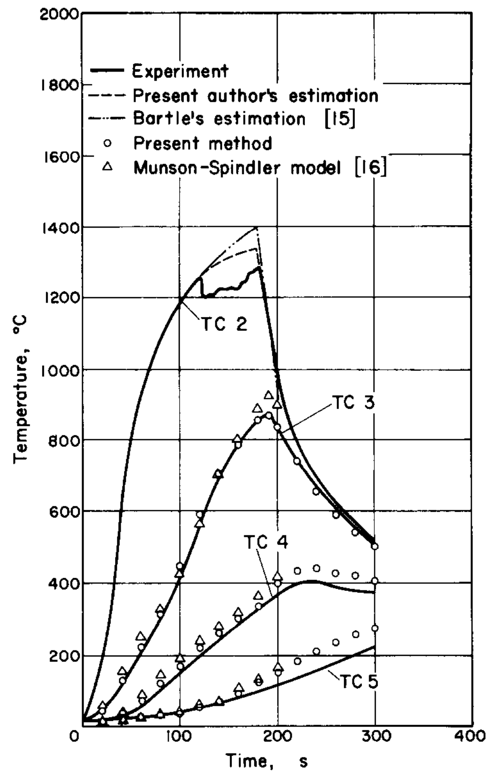

FIG. 5. Temperature histories for specimen 2.

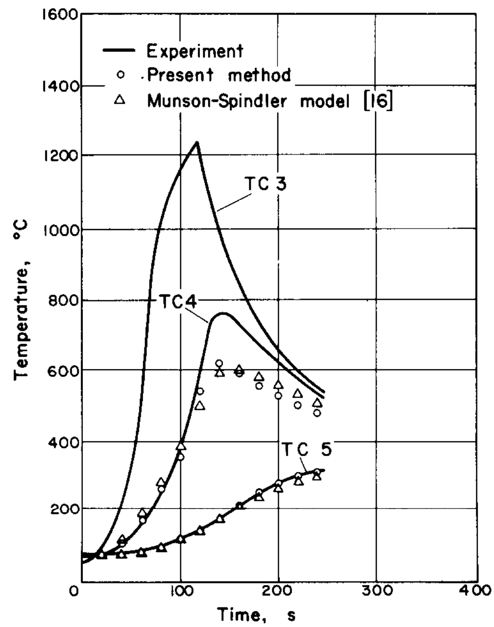

FIG. 6. Temperature histories for specimen 3.

* Because of a noisy signal in the second thermocouple for specimen 2, it was necessary to estimate the experimental temperature for a portion of the test.

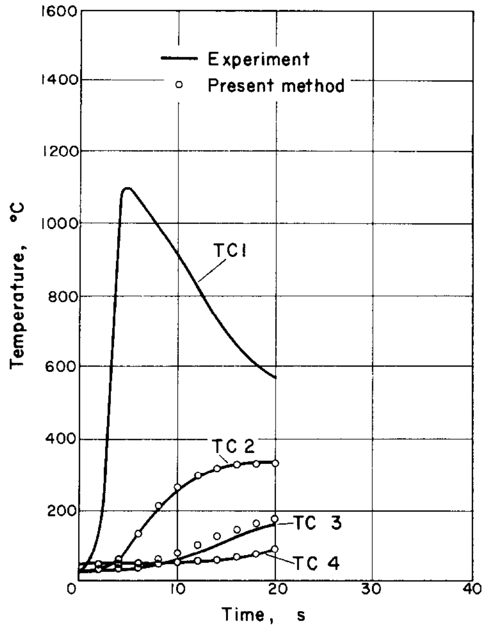

FIG. 7. Temperature histories for specimen 4.

presents the predicted and observed temperature histories for the 10 MW arc supersonic-pipe test; the agreement is excellent in spite of the fact that the properties can be expected to differ slightly from those in Table 2, because of the different fabric layup angle. Predictions from the Munson–Spindler model were not available for this specimen.

One advantage of the nonlinear regression approach is that the root-mean-square (rms) difference between the measured and predicted temperatures may be calculated directly from the sum-of-squares function, equation (3). For the four specimens these rms differences were 27·22°C, 24·17°C, 60·56°C and 12·61°C respectively. These numbers confirm objectively that there was good agreement between the experiment and the authors' model except for specimen three as noted above.

CONCLUSIONS

Nonlinear regression is a practical tool for rapid, accurate, determination of the thermal properties of a charring ablator from transient temperature histories. Figures 4–7 demonstrate that the nonlinear regression properties used in the modified Stefan model describe the carbon–phenolic's behavior over the range of cold-wall heat fluxes from 0·45 to 15·8 MW/m^2 as accurately as conventionally measured properties used in a more complex model. These excellent results were obtained in spite of the fact that the experimental data used to determine the properties were gathered in tests not designed for this purpose. By using tests specifically designed for nonlinear regression analysis, more precise properties could be determined (especially the value of c_g) and the value of c_v would not have to be specified. It must be realized, however, that to use the procedure a large digital computer is required and it is helpful to have an automatic data reduction facility available. Nonlinear regression shows great promise for the simultaneous determination of numerous properties required in complex models, especially when it is difficult to design tests to measure one property at a time. The method appears to offer many possibilities when used with models describing turbulent or statistical type phenomena.

ACKNOWLEDGEMENTS

The authors have received valuable assistance in this work from many associates. Messrs. Henry E. Hoercher and Robert Shaw, Jr. of Avco/ATD made available their unpublished data for the authors' use. Special recognition is due Professor P. L. Thibaut Brian of the Massachusetts Institute of Technology for many helpful suggestions for the finite difference calculations. The continual interest and encouragement of Dr. T. W. Mix is gratefully acknowledged.

REFERENCES

1. J. V. BECK, The optimum analytical design of transient experiments for simultaneous determinations of thermal conductivity and specific heat, Ph.D. Thesis, Michigan State University (1964).
2. J. V. BECK and A. M. DHANAK, Simultaneous determinations of thermal conductivity and specific heat, *Am. Soc. Mech. Engrs* Paper No. 65-HT-14 (1965).
3. J. V. BECK, Transient sensitivity coefficients for the thermal contact conductance, *Int. J. Heat Mass Transfer* **10**, 1615–1617 (1967).
4. J. V. BECK, Analytical determination of high temperature thermal properties of solids using plasma arcs, *Proc. 8th Conf. Thermal Conductivity*, (C. Y. Ho and R. E. TAYLOR, editors), pp. 1009–1030. Plenum Press, New York (1969).

5. R. C. PFAHL, JR., Nonlinear least squares: A method for simultaneous thermal property determination in ablating polymeric materials, *J. Appl. Polym. Sci.* **10**, 1111–1119 (1966).
6. ANON., Avco arc test capabilities, Avco Space Systems Division, Res. and Tech. Laboratories, Wilmington, Mass., AVSSD-148-66-CA (1966).
7. R. C. PFAHL, JR. and D. DROPKIN, Thermocouple temperature perturbations in low-conductivity materials, *Am. Soc. Mech. Engrs* Paper No. 66-WA/HT-8 (1966).
8. J. STEFAN, *Ann. Physik Chem.* **42**, 269 (1891).
9. R. J. GROSH, Transient temperature change in semi-infinite porous solid with phase change and transpiration effects, WADD TR 60-105, Wright–Patterson Air Force Base, Ohio (1960).
10. R. J. BARRIAULT and J. YOS, Analysis of the ablation of plastic heat shields that form a charred surface layer, *ARS Jl* **30**, 823 (1960).
11. W. D. MURRAY and F. LANDIS, Numerical and machine solutions of transient heat-conduction problems involving melting or freezing, *J. Heat Transfer*, **81C**, 106–112 (1959).
12. R. T. SWANN and C. M. PITTMAN, Numerical analysis of the transient response of advanced thermal protection systems for atmospheric entry, NASA TN D-1370 (1962).
13. J. CRANK and P. NICOLSON, A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type. *Proc. Camb. Phil. Soc.* **43**, 50–67 (1947).
14. R. C. PFAHL, JR., An experimental and analytical investigation of the internal mechanisms of ablative heat transfer in charring cork, Ph.D. Thesis, Cornell University, Ithaca, N.Y. (1965).
15. R. S. BARTLE, Private communication, June 9, 1965.
16. T. R. MUNSON and R. J. SPINDLER, The transient thermal behavior of decomposing materials: Part I, general theory and application to convective heating, IAS Paper No. 62–30 (1962).
17. T. R. MUNSON *et al.*, An advanced analytical program for charring ablators, Avco Space Systems Division Report No. AVSSD-0172-67-RR, Vol. I, pp. 91–99 (1967).
18. R. W. FARMER, Thermogravimetry of phenol–formaldehyde polycondensates, Tech. report AFML-TR-65-246, Part II, pp. 6–7 and 15–16 (1967).
19. M. E. WEBER, Improving the accuracy of Crank–Nicolson numerical solutions to the heat-conduction equation, *J. Heat Transfer* **91 C**, 179–191 (1969).

MESURE SIMULTANÉE DE SIX PROPRIÉTÉS THERMIQUES D'UNE MATIÈRE PLASTIQUE EN TRAIN DE SE CARBONISER

Résumé—Six propriétés thermiques d'un matériau en résine phénolique qui se carbonise sont mesurées en une seule expérience. L'expérience, effectuée dans deux installations hyperthermiques à arc de plasma, est décrite et un modèle de transport de chaleur est présenté comme extension du modèle classique de fusion de Stefan. Le modèle est résolu en employant une méthode de différences finies qui a plusieurs caractéristiques uniques. Les propriétés sont calculées à partir des mesures expérimentales en employant le modèle et l'analyse de la régression non linéaire. On montre que les valeurs calculées des propriétés sont en bon accord avec les valeurs obtenues à partir des essais classiques. On démontre par des calculs comparatifs que le modèle modifié de Stefan avec des propriétés de régression non linéaire prédit le comportement thermique transitoire aussi bien qu'un modèle d'ablation plus sophistiqué avec des propriétés mesurées classiquement.

GLEOCHZEITIGE MESSUNG VON SECHS THERMISCHEN EIGENSCHAFTEN EINES VERKOHLENDEN KUNSTSTOFFS.

Zusammenfassung—In einem einzigen Versuch werden sechs thermische Eigenschaften eines verkohlenden Kohlenstoff–Phenol–Kunststoffes gemessen. Der Versuch, der in 2 hyperthermischen Plasma-Lichtbögen durchgeführt wird, ist beschrieben. Ein Wärmeübertragungsmodell wird angegeben, eine Erweiterung des klassischen Diffusionsmodells von Stegan. Das Modell wird durch Anwendung eines Differenzenverfahrens gelöst, das mehrere einmalige Besonderheiten aufweist. Die Stoffwerte werden aus den experimentellen Ergebnissen berechnet, wobei das Modell und eine nichtlineare Regressionsanalyse verwendet werden. Es wird gezeigt, dass die berechneten Stoffwerte mit den Werten aus konventionellen Messungen gut übereinstimmen. Durch Vergleichsrechnungen wird gezeigt, dass das vereinfachte Stefan-Modell mit nichtlinearen Regressionseigenschaften das instationäre thermische Verhalten ebenso voraussagt, wie ein etwas mehr verfälschtes Abschmelzmodell mit konventionell gemessen Stoffwerten.

ОДНОВРЕМЕННОЕ ИЗМЕРЕНИЕ ШЕСТИ ТЕПЛОФИЗИЧЕСКИХ ХАРАКТЕРИСТИК ОБУГЛЕННОГО ПЛАСТИКА

Аннотация—Измеряются одновременно шесть тепловых характерис-тик обугленного углеродно-фенольного материала. Дано описание эксперимента на двух гипертермических дуговых плазменных установках, представлена модель теплопереноса, являющаяся развитием классической модели плавления Стефана. Модель решается с применением метода конечных разностей, который имеет несколько специфических особенностей. Характеристики вычислены, исходя из экспериментальных измерений с использованием модели и анализа нелинейной регрессии. Показано, что полученные значения параметров хорошо согласуются с расчетными данными. Путем сравнительных расчетов сделан вывод, что модифицированная модель Стефана со свойствами нелинейной регрессии описывает неустановившийся тепловой режим аналогично более сложной модели абляции с измеренными параметрами.